jpht
Research Article

Journal of Postharvest Technology
04 (01): 031-036, Jan' 2016
www.jpht.info

Semi-analytical Approach for Prediction of Freezing Times of Regular Shaped Foods

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Received : 26 Nov 2015
Revised : 23 Dec 2015
Accepted : 28 Dec 2015

Keywords
Food freezing time
Planks equation
Pham method
Cooling

Abstract

Food freezing is one of the most significant applications of refrigeration. In order for freezing operations to be cost-effective, it is necessary to optimally design the refrigeration equipment. This requires estimation of the freezing times of foods. Estimation of food freezing time analytically is not possible, and numerous semi analytical methods for predicting food freezing times are available. A new method has been proposed for the prediction of freezing time and the performance of these various methods is evaluated in this paper by comparing their results to experimental freezing time data obtained from the literature. The proposed method shows error ranges upto 10 %, hence may be used for estimation of food freezing time.

INTRODUCTION

It is known that the freezing of food effectively reduces the activity of micro-organisms and enzymes, thus retarding deterioration. In addition, crystallization of water reduces the amount of liquid water in food items and helps in inhibiting the microbial growth (Heldman, 1975).

In order for food freezing operations to be viable commercially, it is necessary to optimally design the refrigeration equipment to fit the specific requirements of the particular freezing application. The design of such refrigeration equipment requires estimation of the freezing times of foods, as well as the corresponding refrigeration loads.

Numerous methods for predicting food freezing times have been proposed. The designer is thus faced with the challenge of selecting an appropriate estimation method from various available methods. This paper focuses upon those methods which are applicable to regularly shaped food items. The performance of these various methods is evaluated by comparing their results to experimental freezing time data obtained from the literature.

The freezing of food is a complex process and before freezing sensible heat must be removed from the food to decrease its temperature from the initial temperature to the initial freezing point of the food. This initial freezing point is somewhat lower than the freezing point of pure water due to dissolved substances in the moisture within the food. At the initial freezing point, a portion of the water within the food crystallizes and the remaining solution becomes more concentrated. Thus, the freezing point of the unfrozen portion of the food is further reduced. As the temperature continues to decrease, the formation of ice crystals increases the concentration of the solutes in solution and depresses the freezing point further. Thus, during the freezing process, the ice and water fractions in the frozen food depend upon temperature. Since the thermophysical properties of ice and liquid water are quite different, the corresponding properties of the frozen food are temperature dependent. Therefore, due to these complexities, it is not possible to derive exact analytical solutions for the freezing times of foods.

Numerical estimates of food freezing times can be obtained using appropriate finite element or finite difference computer programs. However, the effort required to perform this task makes it impractical for the design engineer. In addition, two-dimensional and three-dimensional simulations require time consuming data preparation and significant computing time. Hence, the majority of the research effort to date has been in the development of semi-analytical food freezing time prediction methods which make use of simplifying assumptions.

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ISSN 2348-4330

In the following discussion, the basic freezing time estimation method developed by Plank is discussed first, followed by a discussion of those methods which are based upon modifications of Plank's equation. The discussion then goes to those methods in which the freezing time is calculated as the sum of the precooling, phase change and subcooling times. The next section deals with the proposed method.

Plank's Equation

In the first paper Plank (1913), presented a formula to calculate the freezing time for a block of ice. Different geometric shapes are considered: cylinder, quadratic and rectangular rods as well as slabs. In his second paper (1941) a similar calculation method is used for food products. In this latter paper the following approximations are made:

- the entire foodstuff has freezing temperature right from start
- heat transfer through the foodstuff is by thermal conduction
- physical properties are independent of temperature
- volume change is neglected

It is the most widely known basic method for estimating the freezing times of foods. Plank's freezing time estimation method is given as follows:

Plank's freezing time estimation method is given as follows:

$$t = \frac{\rho_s L}{T_f - T_\infty}\left[\frac{PD}{h} + \frac{RD^2}{k_s}\right] \qquad (1)$$

Where L is latent heat of fusion (J/kg), ρ_s is density of frozen food (kg/m^3), T_f is the initial freezing temperature of the food, T_∞ is the freezing medium temperature, D is the thickness of the slab or the diameter of the sphere or infinite cylinder, *h* is the convective heat transfer coefficient, k_s is the thermal conductivity of the fully frozen food, and P and R are geometric factors. For the infinite slab, P=1/2 and R=1/8. For a sphere, P and R are 1/6 and1/24, respectively, and for an infinite cylinder, P=1/4 and R=1/16.

Various researchers have noted that Plank's method does not accurately predict the freezing times of foods because it assumes that freezing of foods takes place at a constant temperature, and not over a range of temperatures, as is the case in actual food freezing processes. In addition, the thermal conductivity of the frozen food is assumed to be constant, but in reality, the thermal conductivity varies greatly during freezing. Another limitation of Plank's equation is that it neglects the removal of sensible heat above the freezing point. Furthermore, Plank's method is only applicable to infinite slabs, infinite cylinders, and spheres. Researchers have subsequently focused upon development of improved freezing time estimation methods which account for pre-cooling and sub-cooling times, varying thermal properties, irregular geometries, and phase change over a range of temperatures.

Modifications to Plank's Equation

Cleland and Earle (1977 & 1979) improved upon Plank's model by incorporating corrections to account for the removal of sensible heat both above and below the initial freezing point of the food as well as temperature variation during freezing. Regression equations were developed to estimate the geometric parameters, P and R, for infinite slabs, infinite cylinders and spheres. In these regression equations, the effects of surface heat transfer, precooling and final subcooling are accounted for by means of the Biot number, Bi, the Plank number, Pk, and the Stefan number, Ste, respectively. The volumetric latent heat, ρsL , in Plank's equation is replaced with the volumetric enthalpy change of the food, ΔH10, between the freezing temperature, Tf, and the final center temperature, assumed to be -10^{0}C.

Working with fish, Nagaoka, Takaji, and Hohani (1955), use the original Plank equation with the values of P and R for cylinders, plus the definition of an equivalent diameter De, to account for the non-cylindrical transversal shape of the fish. This expression is then multiplied by a factor 0.008.

$$t = (1 + 0.008\ T_i)\frac{\rho_s L}{(T_f - T_\infty)}\left(\frac{D_e^2}{16 k_s} + \frac{D_e}{h}\right) \qquad (2)$$

where $D_e = \frac{2\,ab}{a + b}$

Where a and b are the widest and shortest dimensions of the fish transversal section.

Precooling, Phase Change and Subcooling Time Calculations

Numerous researchers have taken a different approach to account for the effects of sensible heat removal above and below the initial freezing point. In these methods, the total freezing time, t, is the sum of the pre-cooling, phase change and sub-cooling times:

$$t = t_1 + t_2 + t_3 \quad (3)$$

where t1, t2 and t3 are the pre-cooling, phase change and sub-cooling times, respectively.

Pham (1984) also devised a food freezing time estimation method, similar to Plank's equation, in which sensible heat effects are considered by calculating precooling, phase change and subcooling times separately. In addition, Pham suggested the use of a mean freezing point, which is assumed to be 1.5K below the initial freezing point of the food, to account for freezing which takes place over a range of temperatures. Pham's freezing time estimation method is stated in terms of the volume and surface area of the food item and is therefore applicable to food items of any shape. Pham (1984) subsequently simplified the previous freezing time estimation method (1984) to yield a single equation which includes precooling, phase change and subcooling.

Proposed Model

In this paper an analytical based approach has been made for calculating freezing times of irregular shaped foods based upon first term approximation of transient heat transfer analysis and geometric shape factor (E) that is a function of the shape and the Biot number. The analytical expressions for the geometric shape factor are proposed by Hossain et al. (1992) for cooling applications, called the “equivalent heat transfer dimensionality” E to calculate the freezing times of irregularly shaped food items. The freezing time of an irregularly shaped object t_{shape} is related to the freezing time of an infinite slab (t_{slab}) as follows:

$$t_{shape} = \frac{t_{slab}}{E} \quad (4)$$

$t_{slab}=t_1+t_2+t_3$, where

$$t_1 = \frac{D^2}{\alpha V_1^2} \ln\left[\frac{2N^2}{\theta_1 V_1^2 (V_1^2 + N^2 - N)}\right] \quad (5)$$

where, $\theta_1 = (T_f - T_\infty)/(T_i - T_\infty)$

$$t_2 = \frac{\rho_s L}{T_f - T_\infty}\left[\frac{D}{2h} + \frac{D^2}{8k_s}\right] \quad (6)$$

$$t_3 = \frac{D^2}{\alpha V_3^2} \ln\left[\frac{2N^2}{\theta_3 V_3^2 (V_3^2 + N^2 + N)}\right] \quad (7)$$

where, $\theta_3 = (T_F - T_\infty)/(T_f - T_\infty)$

where, N= Biot number, L= actual latent heat (kJ/kg), T_m =temperature of air (^{0} C), T_i = initial temperature (^{0} C), T_f = initial freezing temperature (^{0} C), D = characteristic dimension (m), $\theta = (T - T_\infty)/(T_i - T_\infty)$ V_1, V_3 =first root of transcendental equation $V.\tan(V) = N$ for pre-cooling and sub-cooling time respectively.

Results and Discussion

The developed method alongwith the other analytical methods were compared as per experimental conditions of Kumar (2006) for shrimp under environmental temperatures of -60 C, -100 C, 140 C and centre temperature fixed at -30 C. The experimental conditions for slab shaped, cylindrical shaped and spherical shaped shrimp is shown in tabular form as Table 1, 2 and 3 respectively, where, T_∞ = outside air temperature and T_c = final core temperature.

Table 1 Experimental conditions for Slab Shaped Shrimp

Case	Size	(^{0}C)	T_c (^{0}C)
S1	8 cm 4 cm 2 cm	-140	-30
S2	8 cm 4 cm 1 cm	-140	-30
S3	8 cm 4 cm 2 cm	-100	-30
S4	8 cm 4 cm 1 cm	-100	-30
S5	8 cm 4 cm 2 cm	-60	-30
S6	8 cm 4 cm 1 cm	-60	-30

Table 2 Experimental conditions for Cylinder Shaped Shrimp

Case	R(cm)	L(cm)	(^{0}C)	T_F (^{0}C)
C1	2	8	-140	-30
C2	2.5	10	-140	-30
C3	2	8	-100	-30
C4	2.5	10	-100	-30
C5	2	8	-60	-30
C6	2.5	10	-60	-30

Table 3 Experimental conditions for Spherical Shaped Shrimp

Case	D (cm)	T_a (^{0}C)	T_F (^{0}C)
SP1	5	-140	-30
SP2	5	-100	-30
SP3	5	-60	-30
SP4	6	-140	-30
SP5	6	-100	-30
SP6	6	-60	-30

Performance of Food Freezing Time Estimation Methods

Table 4 summarizes the experimental time and predicted time as proposed by Modified Plank (1955), Cleland et. al. (1984), Pham (1986) and present model, while Table 5 represents for cylindrical shaped food and Table 6 represents for spherical shaped foods.

Table 4 Comparison of Results in case of Slab Shaped Food Material

Case	Experimental time (Min)	Modified Plank (1955) (Min)	Cleland et al. (1984) (Min)	Pham (1986) (Min)	Proposed Method (Min)
S1	12.1	11.9	11.8	13.2	13.3
S2	7.0	6.56	6.4	8.0	7.5
S3	16.8	16.5	17.1	17.3	17.2
S4	9.2	9.2	9.4	9.9	9.9
S5	27.2	28.5	28.9	29.2	28.9
S6	15.6	15.6	15.9	16.4	17.0

Table 5 Comparison of Results in case of Cylindrical Shaped Food Material

Case	Experimental time (min)	Modified Plank (1955) (min)	Cleland et al. (1984) (min)	Pham (1986) (min)	Proposed Method (Min)
C1	18.2	17.6	17.9	19.1	18.9
C2	24.2	22.4	24.5	24.6	26.1
C3	25.3	24.8	25.8	25.5	26.1
C4	33.7	31.8	34.2	34.1	33.6
C5	42.5	40.2	43.6	43.2	43.4
C6	56.6	53.2	56.6	57.8	60.1

Table. 6 Comparison of Results in case of Sphere Shaped Food Product

Case	Experimental time (min)	Modified Plank (1955) (min)	Cleland et al. (1984) (min)	Pham (1986) (min)	Proposed Method (Min)
SP1	17.6	15.1	15.8	18.1	19.1
SP2	23.4	21.6	23.6	24.0	24.5
SP3	38.5	34.2	40.4	42.1	41.1
SP4	22.5	20.6	23.6	23.8	24.7
SP5	30.5	26.8	30.7	32.8	32.9
SP6	49.3	42.9	50.1	51.9	52.5

The values of errors associated with the different methods in comparison to experimental ones is mentioned in Tables 7, 8 and 9 for slab, cylindrical (L = 2D) and spherical shaped shrimps respectively.

Table 7 Percent Error associated with Different Methods for Freezing Time in case of Slab Shaped Shrimp

Case	Modified Plank (1955)	Cleland et al. (1984)	Pham (1986)	Proposed Method
S1	-1.6	-2.4	9.1	9.9
S2	-6.2	-8.5	14.2	7.1
S3	1.8	1.7	2.9	8.9
S4	0	2.1	7.6	7.6
S5	4.7	6.2	7.3	6.2
S6	0	1.9	5.1	8.9

In case of finite slab shaped food, the modified Plank's equation predicted the freezing time with better accuracy for maximum cases, in comparison to others; although the error percentage range was more. The error percentage using modified Plank's equation was varying between -6.2 to 4.7 %, Cleland et al. method shows a variation between -8.5 to 6.2 %, Pham method predicted the freezing time with error percentage varying between 2.9 to 14.2%, and proposed method shows error variation between 6.2 to 9.9 %. The error percentage ranges thus are 10.9 %, 14.7 %, 11.3 %, 2.7 % and 7.2 % for modified Plank's method, Cleland et al. method, Pham method, and proposed method respectively. Also we find that the error percentage range for the proposed method is the minimum.

Table 8 Percent Error associated with Different Methods for Freezing Time in case of Cylinder Shaped Shrimp

Case	Modified Plank (1955)	Cleland et al. (1984)	Pham (1986)	Proposed Method
C1	-3.2	-1.6	4.9	3.8
C2	-7.4	1.2	1.6	7.8
C3	-1.9	1.9	0.8	3.1
C4	-5.6	1.4	1.1	-0.2
C5	-5.4	2.5	1.6	2.1
C6	-6.0	0	2.1	6.1

In case of finite cylindrical shaped food, the modified Cleland et al. method predicted the freezing time with better accuracy, in comparison to others for most of the cases. The error percentage using Cleland et al. method was varying between -1.6 to 2.5 %, modified Plank's. method shows a variation between -3.2 to -7.4 %, Pham method predicted the freezing time with error percentage varying between 0.8 to 2.1 %, and proposed method shows error variation between -0.2 to 7.8 %. The error percentage ranges thus are 4.2 %, 4.1 %, 1.3 % and 8.0 %for modified Plank's method, Cleland et al. method, Pham method and proposed method respectively.

Table 9 Percent Error associated with Different Methods for Freezing Time in case of Spherical Shaped Shrimp

Case	Modified Plank (1955)	Cleland et al. (1984)	Pham (1986)	Proposed Method (Min)
SP1	-14.2	-10.2	2.8	8.5
SP2	-7.6	0.8	2.5	4.7
SP3	-11.9	4.9	9.3	6.7
SP4	-8.4	4.8	5.7	9.7
SP5	-12.1	0.6	7.5	7.8
SP6	-12.9	1.6	5.2	6.5

In case of spherical shaped food, the modified Cleland et al. method predicted the freezing time with better accuracy, in comparison to others for most of the cases. The error percentage using Cleland et al. method was varying between -10.2 to 0.8 %, modified Plank's. method shows a variation between -7.6 to -14.2 %, Pham method predicted the freezing time with error percentage varying between 2.5 to 9.3 %, and proposed method shows error variation between 4.7 to 9.7 %. The error percentage ranges thus are 11 %, 6.6 %, 6.8 % and 5 % for modified Plank's method, Cleland et al. method, Pham method and proposed method respectively.

The differences in actual and predicted values of freezing time may be attributed to inaccurate assumptions made for predicting the freezing time and the approximate method used for the estimation of thermophysical properties. The developed methods predicted the freezing time with errors associated with them is nowhere more than 10%. This shows that the models are accurate enough for predicting the freezing times.

CONCLUSION

A new method for the prediction of freezing times has been proposed for food items. The average prediction error for all the three cases of slab, cylinder and spherical geometries of foods were under 10 %, hence the proposed model may be used for prediction of food freezing times.

NOMENCLATURE

L : latent heat of fusion (J/kg)
ρ_s : density of frozen food (kg/m^3)
T_f : initial freezing temperature of the food
T_∞ : freezing medium temperature
D: thickness of the slab or the diameter of the sphere or infinite cylinder
H: convective heat transfer coefficient
k_s : thermal conductivity of the fully frozen food
P & R :Plank's geometric factors
a & b : widest and shortest dimensions of the fish transversal section respectively
t_1,t_2 & t_3 : pre-cooling, phase change and sub-cooling time respectively
t : freezing time
TF : final center temperature of food item
Tf : initial freezing temperature of food item
Ti : initial temperature of food item
T_∞ : freezing medium temperature

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